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## Introduction

Since the early work of Besant and Rayleigh on the collapse of a vacuum bubble in an ideal liquid a great many investigations have been undertaken on cavitation and spherical growth problems within the general framework of problems related to the collapse of a spherical bubble (see the surveys of Hsieh [1] and Zababakhin [2]). The bulk of the research done on collapse of a bubble in a liquid has been of a theoretical-analytical character. The experimental investigation of bubble collapse problems is hampered by the difficulties of producing single spherical bubbles in an effectively unbounded volume. Techniques are described in the literature for obtaining a single bubble in a liquid by means of electrical discharges, but the experimental work of Gibson [3] shows that electrical discharge is impractical for obtaining a kinetic model of a spherical bubble, due to the complexity of the energy-release process in the discharge gap and due to the presence of bounding surfaces, namely, electrodes, which automatically render the boundary unstable and the bubble collapse process asymmetrical.

Consequently, at the present epoch in bubble collapse research a certain rift exists between theory and experiment and their mutual correlation. A methodological breakthrough in research on cavitation problems has emerged in recent years with the advent of experimental laser techniques. For example, by focusing a single laser pulse in a liquid it is possible to generate the conditions necessary for testing of the theoretical calculations; to obtain a single bubble in an effectively unbounded volume; to obtain $n$ bubbles with predetermined relative phases of the pulsation periods and different relative maximum sizes; and to realize the unique conditions of interaction between a pulsating bubble and a rigid or free surface $[4,5]$.

Experimental Procedure
Collapsing single bubbles are created in a liquid by focusing a single radiation pulse emitted by a ruby laser in water and in viscous liquids (Vaseline oil and glycerin); single bubbles with radii up to 10 mm se investigated. The bubble-collapse kinetics is investigated by shadowgraphic techniques in the frame-by-frame and continuous modes, using a high-speed camera according to the procedure described in [4].

The pressure pulses from the collapsing bubble are measured by means of barium titanate and natural tourmaline pressure pickups with a resolving power of $0.1 \mu \mathrm{sec}$, according to the procedure described in [6]. The pressure pulses are recorded on an OK-17M oscilloscope with a high-resistance preamplifier.

## Experimental Results

One of the preliminary indices of the collapse of a pulsating bubble is the energy loss characteristic of the bubble between successive pulsations; it is readily determined in practice by measuring the bubble pulsation period.

The following relation is valid for the case of a nonascending bubble at a constant external pressure in an unbounded liquid [7]:

$$
E_{r}^{(n+1)} / E_{r}^{(n)}=\left(T_{n+1} / T_{n}\right)^{3}
$$

where $\mathrm{E}_{r}^{(\mathrm{n})}$ is the energy stored by the bubble in the n -th pulsation and $\mathrm{T}_{\mathrm{n}}$ is the period of the n -th pulsation.
Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 109-117, July-August, 1976. Original article submitted June 26, 1975.

[^0]TABLE 1

| Bubble-formation technique | $E_{r}^{(2)} / E_{r}^{(1)}$ | $E_{r}^{(3) / E_{r}^{(2)}}$ |
| :--- | :---: | :---: |
| Detonation of chemical high <br> explosives in water [7] | 0,34 <br> Electrical discharge in water | $0,05-0,01$ |
| Laser breakdown in water |  |  |
| Laser breakdown in glycerin | $0,07-0,01$ | 0,54 |
| Laser breakdown in |  |  |
| Vaseline oil |  |  |



Fig. 1

Table 1 lists comparative data on the parameters characterizing the relative expenditure of energy stored by the bubble for various techniques of creating pulsating bubbles.

It is customarily assumed in accordance with [7] that the main fraction of the energy $\mathrm{Er}^{(n)}$ stored in a bubble goes for the emission of acoustic-shock radiation $\mathrm{E}_{\mathrm{p}}^{(\mathrm{n})}$ and energy input to the next pulsation $\mathrm{E}_{\mathrm{r}}^{(\mathrm{n}+1)}$ :

$$
E_{r}^{(n)}=E_{p}^{(n)} \div E_{r}^{(n+1)}+q .
$$

The energy losses $q$ are neglected, so that it is normally assumed in practice that the energy difference between two successive pulsations corresponds to the energy of acoustic-shock radiation from the collapsing cavity.

As noted in [4], however, the loss of energy from a pulsating bubble is attributable not only to acousticshock radiation from the cavity, but also to asymmetrical collapse, which causes the bubble to disintegrate more rapidly in subsequent pulsations.

## Experiments with Water

To gain a more detailed understanding of the given process and the relationship of the preliminary data published in [4] to the theoretical investigations [8-10] we have measured the pressure pulses from a collapsing bubble in water in correlation with motion-picture studies.

Representative oscillograms of pressure pulses from a collapsing bubble in water are given in Fig. 1a-c. Figure 1a corresponds to acoustic-shock radiation from an almost-symmetrically collapsing bubble, while Fig。1b and c represent situations in which asymmetrical collapse is observed, the former for asymmetry in the form of loss of stability in the vicinity of minimum bubble size and the latter for the case of pronounced asymmetrical collapse of the bubble. An example of asymmetrical bubble collapse of this type is given in the film strip of Fig. 2, in which it is seen that prior to the instant at which the bubble has minimum diameter cumulative jets shoot through it as a result of its asymmetry at the instant of maximum diameter. This kind of collapse causes the bubble to disintegrate rapidly.

Qualitative and quantitative analyses of the film strips and measured pressure pulses from a collapsing bubble show that in the case of asymmetrical collapse the pressure pickup records "precursors," i.e., pressure pulses (indicated by arrows in Fig. 1b and c) that precede the main pulse emitted by the bubble during collapse. When such precursors are present, the slope of the amplitude rise of the main pressure pulse is observed to decrease and the maximum value of the pressure amplitude to decrease by more than $50 \%$ (for a relative measurement error of $\pm 7 \%$ ).

Figure 3 gives the results of measurements of the maximum pressure amplitudes p recorded at a distance $r=1.5 \mathrm{~cm}$ from the breakdown center for various bubbles. Recognizing that the given investigation includes



Fig. 3
bubbles that deviate from a spherical configuration, we adopt as the maximum bubble radius (see Fig. 3) the quantity $R_{6}$ calculated in terms of the pulsation period $T_{1}$ according to the Willis equation $R_{0}=\left(T_{1} / 1.83\right) \times$ $\sqrt{p_{0} / \rho}$, which is valid for spherical bubbles from laser breakdown within the error limits ( $\pm 1 \%$ ) of the measurement of $R_{0}$ and $T_{1}$ 。

The results can be approximated by the equation

$$
p=K\left(R_{0} / r\right)^{\alpha},
$$

in which $p$ is the maximum pressure in the wave (bar); $R_{C}$ is the maximum bubble radius; and $r$ is the distance to the pressure measurement point.

For the given experiments we have $\alpha=1.2$ to 1.3 . The dimensional factor K lies between the limits 90 and $120 \mathrm{bar}, \mathrm{i}_{\mathrm{o}} \mathrm{e}_{\mathrm{o}}$, for the upper limits of the amplitudes corresponding to nearly symmetrical collapse, $\mathrm{K}=120$ bar. For the oscillograms without precursors we calculate the acoustic energy radiated by the bubble in collapse. The calculations are carried out according to the equation [7]

$$
E_{p}^{(n)}=\frac{4 \pi r^{2}}{\rho c} \int_{0}^{7 \theta_{\mathbf{2}}} p^{2}(t) d t+\frac{4 \pi r^{2}}{\rho c} \int_{0}^{7 \theta_{2}} p^{2}(t) d t,
$$

in which $r$ is the distance from the pickup; $\rho c$ is the wave impedance of the liquid; and $\theta_{1}, \theta_{2}$ are the time constants of the exponential decay of the left and right branches of the pulse.

The results of processing oscillograms without precursors show that the bubble radiates as sound on the average $75 \%$ (for an average total measurement error of $\pm 24 \%$ ) of the stored energy $E_{r}^{(n)}=4 / 3 \pi R_{0}^{3} p_{0}$ of the bubble.

We processed some of the oscillograms with precursors only for the main pulse corresponding to bubble collapse. Processing of such oscillograms without consideration for the precursor peaks against the background of the main pulse show that the energy fraction associated with the main pulse decreases on the average to $50 \%$ (for a measurement error of $\pm 34 \%$ ) relative to the stored energy $\mathrm{E}_{\mathrm{r}}^{(\mathrm{n})}$ in the bubble.

The pulses from precursors are ignored due to the increasing error of measurement of $p$ and $\theta$, so we are unable to estimate with sufficient accuracy the energy radiated by the precursors. We can merely point out that in some experiments the precursor amplitude attains almost half the amplitude of the "attenuated" main pulse from a collapsing bubble, or about one fourth of the pulse amplitude for a symmetrically collapsing bubble.

Experiments with Vaseline Oil and Glycerin
To determine the role of viscosity in the bubble collapse problem we have investigated collapse in Vaseline oil with a density $\rho=0.868 \mathrm{~g} / \mathrm{cm}^{3}$ and viscosity $\nu=1.3 \cdot 10^{3} \mathrm{cP}$ at $\mathrm{t}^{\circ}=22^{\circ} \mathrm{C}$ and in glycerin with a density $\rho=$


Fig。 4
$1.26 \mathrm{~g} / \mathrm{cm}^{3}$ and viscosity $\nu=2 \cdot 10^{3} \mathrm{cP}$ at $22^{\circ} \mathrm{C}$ ，using the same experimental arrangement and conditions．The results of the oscillographic measurements show that the amplitude of pulses radiated in the collapse of a bub－ ble in the given viscous liquids decreases by $1 / 5$ to $1 / 8$ relative to the values measured for water（see Fig．3）． The qualitative structure of the pulses is close to that of the pulses from a symmetrically collapsing bubble in water．Precursors are not observed in glycerin and Vaseline Oil，despite the fact that conditions are created for asymmetrical collapse．

Figure 1d gives a typical oscillogram of a pressure pulse from a pulsating bubble in Vaseline oil，and Fig。 4 gives a typical film strip for Vaseline oil，showing that loss of stability occurs somewhere near the in－ stant of minimum bubble radius and the liquid retains this state until the maximum radius is attained by the bubble in the second pulsation，whereupon the bubble acquires a regular sperical shape and pulsates stably un－ til all its energy is lost．

The observed pattern can be explained as follows．Loss of stability on the part of a bubble collapsing in a viscous liquid can occur as a result of the reduction in viscosity over the bubble boundary due to the increase in temperature of the gas in the bubble interior．In expansion，on the other hand，the walls are cooled，the viscosity increases，and the instabilities are eradicated by the viscosity effect．

It follows from the oscillographic and motion－picture data that the pulsations are recurrent and stable in viscous liquids．Consistent stability of the pulsations is observed for viscous liquids，i．e．，even if bubble asym－ metry is created，it vanishes in the subsequent pulsation stages（see Fig。4），so that recurrent stable pulsations are generated．

The reason that precursors are not observed in viscous liquids clearly lies in the considerably smallex velocities of the cumulative jets，because precursors are the results of hydraulic impact of the cumulative jets against the opposite wall of the bubble，with a local pressure at the impact point $p_{L}=\rho c u / 2$（ $u$ is the jet velocity at impact）．

Despite the fact that the local impact pressures $\mathrm{pL}_{\mathrm{L}}$ can attain the same orders of magnitude（ $\sim 10^{3} \mathrm{bar}$ ）as in bubble collapse，the acoustic emission from the impact source is insignificant；indirect estimates（based on the fraction of the main radiation from precursors and without precursors）indicate that it cannot exceed $1 / 4$ of the bubble energy．This fact explains why it is methodologically difficult to record the precursor parameters in terms of absolute values．

## Discussion

The anomalous increase in the pressure amplitude in pulsations of a gas bubble near a free surface is de－ scribed in［7］，but a definitive explanation of the observed＂peaks＂in the pulsation oscillograms is not given．


Fig. 5


Fig。 6
It can be definitely concluded on the basis of present-day notions [11] concerning the processes attending underwater explosions near a free surface and the results of our investigations that the "peaks" described in [7] and the "precursors" described here have exactly the same nature, i.e., acoustic radiation from the impact of cumulative jets against the opposite wall of the bubble. Figure 5 shows a film strip of an asymmetrically collapsing bubble, where the asymmetry is determined by the interaction of the pulsating bubble with a free surface [11]. The film clearly exhibits the cumulative jet inside the bubble. For large indentations, however, the cumulative jet is not as distinctly observed, because it appears at later stages of the collapse process (see Fig. 4). The bubble "senses" a free surface and a rigid surface up to a distance $5 \mathrm{R}_{0}$, $\mathrm{i}_{\circ} \mathrm{e}$., if the bubble pulsates at a distance less than $5 \mathrm{R}_{0}$ from a free or a rigid surface, collapse asymmetry is witnessed. This consideration enables us to approximately determine the adequacy of the unbounded-volume assumption in setting up experiments on the symmetrical collapse of bubbles in a liquid. The latter result further confirms the conclusion that electrical discharge is totally impractical for investigations of spherical collapse. Moreover, as the experience of the present investigations reveal, laser breakdown in liquids makes it possible to realize conditions sufficiently close to those of symmetrical collapse. Figure 6 shows a film strip of symmetrical collapse. However, the indicated adequacy of the unbounded-volume assumption is determined within the resolution limits of the film (to $10^{-2} \mathrm{~cm}$ ) and the time resolution of the piezoelectric transducers (to $10^{-7} \mathrm{sec}$ ), so in principle the linear scale of the volume cannot be greater than $5 \mathrm{R}_{0}$. It is important to bear in mind in this connection that almost-symmetrical collapse of a bubble in water has a statistical characterinsofar as the stability of a bubble collapsing in water depends on a number of other additional factors such as, for example, the purity of the liquid and its gas content. Consequently, comparisons with theoretical studies of the parameters of acoustic radiation from a collapsing spherical bubble [8-10] must be made according to the upper values of the pulsation pressure.

From the water experiments we obtain the dependence of the maximum wave pressures $p$ on the maximum bubble radius $\mathrm{R}_{0}$ and the distance $r$ from the measurement point:

$$
\begin{equation*}
p \simeq 120\left(R_{0} / r\right)^{1.25} . \tag{1}
\end{equation*}
$$

From the results of theoretical calculations [8, 9] for a bubble with an initial maximum radius $\mathrm{R}_{0}$ and initial pressure $\mathrm{p}_{\mathrm{g}}=10^{-3} \mathrm{~atm}$ in its interior we have

$$
\begin{equation*}
p \simeq 400\left(R_{0} / r\right) \tag{2}
\end{equation*}
$$

The difference between the resulting dependences (1) and (2) is mainly attributable to the difference in the "initial" pressures of the gas at the maximum bubble radius, because, judging from the results of an estimation of the ratio $\eta$ of conversion of bubble energy into acoustic radiation energy ( $\eta=\mathrm{E}_{\mathrm{p}} / \mathrm{E}_{\mathrm{r}}$ ), according to the data of [9] we are working in the interval of initial pressures in the bubble from $10^{-2}$ to $10^{-3}$ atm. Furthermore, according to the motion-picture results we have determined the maximum velocities of the bubble boundary in collapse ( $u=530 \mathrm{~m} / \mathrm{sec}, \mathrm{M}=\mathrm{u} / \mathrm{c}=0.33$ ), which fall within the interval of agreement with the results of calculam tions [9,10] based on the conversion parameter $\eta$ as a function of the maximum Mach number M.

The comparative analysis of theory and experiment leads to the conclusion that the theory developed for inviscid liquids is consistent with the present experiments. As a result, the vapor and gas pressure $\mathrm{p}_{\mathrm{g}}$ at the maximum bubble radius $\mathrm{R}_{0}$ in water for the given experiments comes to an average of $4 \cdot 10^{-3} \mathrm{~atm}$ 。

For bubbles produced by laser breakdown in Vaseline oil and glycerin the value of $\mathrm{p}_{\mathrm{g}}$ at $\mathrm{R}_{0}$ is greater than $10^{-2}$ atm. This fact is inferred from an approximate estimate of the conversion parameter $\eta$ and the parameters of Table 1. A reduction in the viscosity by three orders of magnitude does not yield an appreciable increase in the parameter $\eta$. The high gas content is explained by the effective decomposition of glycerin and Vaseline oil into gaseous products as a result of optical breakdown. The indicated singularity prevents one from clarifying more definitively the role of viscosity with regard to the constitution of a bubble in a viscous liquid [2].

Taking into account the experimental data and the results of [9] on the dependence of the parameter $\eta$ on M and $\mathrm{p}_{\mathrm{g}}$, it would seem logical to adopt the parameter $\eta$ as an experimental adequacy criterion for the unboundedness of spherical growth [2] for bubbles in a liquid. Unbounded growth can be approached as $\eta \rightarrow 1(M \rightarrow \infty)$, while for $\eta<1$ spherical growth is precluded by the viscosity of the liquid, the gas content of the bubble, and instabilities.

The sensibility of testing experimentally the unboundedness of spherical growth of bubbles according to the criterion $\eta$ is governed by the complexity of recording by optical film techniques the velocity of the bubble boundary in the vicinity of a singul arity due to diffraction effects induced by the attendant acoustic-shock radiation. We summarize with the following conclusions.

1. It has been shown in the example of bubble collapse from laser breakdown in liquids that the method of generating collapsing bubbles by means of laser breakdown in liquids is more sophisticated than the electrical discharge method.
2. The observed singularities in the spectrum of acoustic-shock radiation from a collapsing bubble are attributable to bubble collapse asymmetry. Asymmetrical and unstable bubble collapse lower the acoustic radiation amplitudes.
3. It has been shown that the viscosity of the liquid can eradicate instability at the boundary of a collapsing bubble, producing stable recurrent bubble pulsations. Viscosity significantly lowers the acoustic radiation amplitudes associated with a collapsing bubble.
4. For a sufficiently symmetrically coll apsing bubble in water the experimental results are consistent with the theoretical developments of [8-10]. The ratio of conversion of bubble energy into acoustic radiation energy attains $90 \%$ in the case of the experimental data for water.
5. It follows from the stated criterion of boundedness of spherical growth and the foregoing experimental results that unbounded spherical growth is largely precluded by the gas content of the bubble and instability of the bubble boundary in the vicinity of a singularity.

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## LITERATURE CITED

1. D.-Y. Hsieh, "Some analytical aspects of bubble dynamics," Trans. ASME, Ser. D: J. Basic Eng., 87, No. 4, 991 (1965).
2. E. I. Zababakhin, "Unbounded growth effects," in: Mechanics in the USSR in the Last Fifty Years [in Russian], Vol. 2, Nauka, Moscow (1970), p. 313.
3. D. C. Gibson, "The kinetic and thermal expansion of vapor bubbles"; "The pulsation time of spark-induced vapor bubbles," Trans. ASME, Ser. D: J. Basic Eng., 94, No, 1, 101; 273 (1972).
4. A. A. Buzukov, Yu. A. Popov, and V. S. Teslenko, "Experimental study of the explosive process induced by focusing of a single laser pulse in water," Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 17 (1969).

5．W．Lauterhorn，＂Kavitation durch Laserlicht，＂Acustica，31，No． 2 （1974）．
6．Yu．E．Nesterikhin and R．I．Soloukhin，High－Speed Measurement Techniques in Gas Dynamics and Plas－ ma Physics［in Russian］，Nauka，Moscow（1967）．
7．R．H．Cole，Underwater Explosions，Princeton University Press，Princeton，New Jersey（1948）。
8．R．Hickling and M．S．Plesset，＂Collapse and rebound of a spherical bubble in water，＂Phys．Fluids，7， No．1， 7 （1964）．
9．V．P．Morozov，＂Numerical analysis of sound radiation by a sperical cavity，＂Tr．Akust．Inst．，No．7， 115 （1969）．
10．A．D．Pernik，＂Cavitation noise，＂Tr．Akust．Inst．，No．7， 87 （1969）．
11．V．K．Kedrinskii，＂Underwater explosion near a free surface，＂Dokl。Akad。Nauk SSSR，212，No．2， 324 （1973）．

ONE－DIMENSIONAL FLOWS AND ISOMAGNETIC SHOCK WAVES
IN A MAGNETIZED CONDUCTING MEDIUM
G．A．Shaposhnikova
UDC 538.4
§1．Consider the one－dimensional flow of an ideally conducting gas containing no space charge，which moves in a current tube in an electromagnetic field；the gas is inviscid and of zero thermal conductivity．For simplicity，we assume that the electric field $E$ and magnetic field $H$ are mutually perpendicular and lie in a plane perpendicular to the direction of motion．The equations for the flow take the following form［1，2］：

$$
\begin{gather*}
\rho u=m=\mathrm{const} ;  \tag{1.1}\\
\rho u \frac{d u}{d x}+\frac{d p}{d x}=\frac{1}{c} j B-\frac{1}{4 \pi} \int_{0}^{H}\left(\mu-\rho\left(\frac{\partial \mu}{\partial \rho}\right)_{T, H}\right) H d H+\frac{1}{4 \pi} B \frac{d H}{d x} ;  \tag{1.2}\\
\rho u\left(c_{p} \frac{d T}{d x}+u \frac{d u}{d x}\right)+\rho u \frac{d}{d x}\left[\frac{1}{4 \pi \rho} \int_{0}^{H}\left(T\left(\frac{\partial \mu}{\partial T}\right)_{\rho, H}-\rho\left(\frac{\partial \mu}{\partial \rho}\right)_{T, H}\right) H d H\right]=E j ;  \tag{1.3}\\
v_{m} d H / d x=u B-c E, p=R \rho T, B=\mu H, j=\sigma(E-u B / c)  \tag{1.4}\\
d E / d x=0,
\end{gather*}
$$

where the symbols are those commonly employed，with $\nu_{m}=\mathrm{c}^{2 / 4 \pi} \sigma$ the magnetic viscosity；the magnetic sus－ ceptibility is defined by Langevin＇s formula［3］：

$$
\begin{equation*}
!=1+\left(4 \pi m_{I I}{ }^{\rho} / M H\right)(\operatorname{cth} \psi-1 / \psi), \psi=m_{\mathrm{H}} H / k T \tag{1.5}
\end{equation*}
$$

where $m_{H}$ and $M$ are the magnetic moment and mass of one molecule of the perfect gas．Then（1．5）gives （1．2）and（ 1.3 ）the form

$$
\begin{gather*}
\rho u d u d x+d p / d x=(1 / c) \sigma\left(E-u B^{\prime} c\right) B+(1 / 4 \pi)(\mu-1) H d H / d x  \tag{1.6}\\
\rho u\left(c_{p} d T / d x+u d u / d x\right)-\rho u(d / d x)\left[\left(k^{\prime} T / M\right)(\psi c t h \psi-1)\right]=\sigma\left(E-u B^{\prime} c\right) E . \tag{1.7}
\end{gather*}
$$

The method of［4］gives us expressions for the changes in speed and Mach number $M$ along the current tube in terms of the flow parameters from（1．1），（1．4），（1．7），and the first two equations in（1．6）：

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